

# Teacher Subject Matter Knowledge of Number Sense

Hannah Briand-Newman

*University of Sydney*

<hnew7092@uni.sydney.edu.au>

Monica Wong

*Australian Catholic University*

<monica.wong@acu.edu.au>

David Evans

*University of Sydney*

<david.evans@sydney.edu.au>

Pedagogical content knowledge has been widely acknowledged by researchers and practitioners as a significant factor for improving student knowledge, understanding and achievement. Recently, the knowledge teachers need for teaching has expanded to include teacher horizon content knowledge, “an awareness of how mathematical topics are related over the span of mathematics included in the curriculum” (Ball, Thames, & Phelps, 2008, p. 403). This study uses a collective case study design, in which three Kindergarten teachers from Greig Heights Primary School participated in a professional learning and development program designed to enhance aspects of their teacher knowledge. This paper will provide an emerging description of the nature of teacher knowledge, and discuss the potential implications this has for catering for the needs of students at-risk of experiencing difficulties in acquiring early numeracy skills (i.e., number sense knowledge).

The acquisition of number sense has been recognised as a fundamental component of learning mathematics. Number sense, or the capacity to make sense of numbers and magnitude, is seen as foundational knowledge needed by children to understand and link quantities to our number system, numerical constructs and mathematical strategies. Children with number sense have developed “meaning” for numbers and their relationships. They are able to recognise the magnitude of those numbers and the “effects of operating” on numbers, and developing “referents” for quantities and measures (Sowder & Klein, 1993, p. 41).

Recent research has confirmed the importance of number sense of young children as they enter school and its association with students’ mathematics achievement in later years (Doig, McCrae, & Rowe, 2003; Gersten & Chard, 1999; Griffin, 2004; Griffin, Case, & Siegler, 1994; Jordan, Kaplan, Ramineni, & Locuniak, 2009). Students commence their first year of school with varying levels of number sense. Students with limited number sense are characterised as developing early counting skills or strategies, relying on procedural knowledge to meet the demands of numeracy in their environment, and having difficulty managing the demands of mathematical language (Evans, Strnadová & Wong, 2007). Number sense therefore needs to be recognised, understood, accommodated and taught effectively. Teacher knowledge of number sense is key to delivering quality early numeracy programs that prevent lasting difficulties with essential skills in our society.

## Knowledge for Teaching

The foundational work of Shulman (1986, 1987) provides a conceptual framework and set of analytic distinctions to the knowledge needed for effective teaching. This framework incorporates seven categories of teacher knowledge across the curriculum: general pedagogical knowledge, knowledge of learners, knowledge of educational contexts, knowledge of the purpose of education, content knowledge, curriculum knowledge and pedagogical content knowledge.

Over the past two decades, research into teacher knowledge has focussed on two overlapping and interdependent domains: pedagogical content knowledge and content

knowledge. Good teachers know both content and “how to get it across” to the students they are teaching (Ball & Hill, 2009, p. 68). However, what this knowledge consists of, and whether other types of knowledge are needed is still being debated.

The knowledge teachers require to teach mathematics is “partly domain specific rather than a single construct of general factors such as a teacher’s overall intelligence, mathematical or teaching ability” (Hill, Schilling & Ball, 2004, p. 27). Hill et al. concluded that teacher knowledge of mathematics for teaching is multidimensional and consists of both general knowledge of content and more specific domain knowledge (i.e., specialised content knowledge). The latter is considered a deeper knowledge of specific content such as number and operations, including knowledge of student misconceptions, analysing unusual procedures or algorithms and providing explanations for rules.

The research of Ball et al. (2008) has culminated in the conceptualisation of the “Mathematical Knowledge for Teaching” model shown in Figure 1. This model refines the initial categories of pedagogical content knowledge and content knowledge proposed by Shulman (1986). Subject matter knowledge depicted in Figure 1 includes common content knowledge, specialised content knowledge, with the addition of horizon content knowledge. Ball et al. (2008) speculate that teachers need horizon content knowledge to be able to see the connections within the mathematics they are teaching. Further, it relates to the mathematics being taught and how it connects to the learning horizon, “...the sort of understanding that gives teachers peripheral vision for where they are and where their pupils are heading, to be conscious of the consequences of how ideas are represented, or the later development that is or possibly impeded – by decisions with the current work” (Ball et al., 2009, p. 98).

Common content knowledge on the other hand represents teacher’s knowledge of the mathematics they teach, including the use of correct mathematical terms and notation, and recognise when students give an incorrect answer. This type of knowledge implies that teachers “must be able to do the work that they assign to their students” (Ball et al., 2008, p. 399), including an insight into the mathematical understanding involved, and how this knowledge links to higher order material. Teachers need to recognise an incorrect answer, whereas knowing the nature of the error is specialised content knowledge.

Beswick, Callingham and Watson (2011) extended the work of Shulman (1987) to include measuring the impact of certain aspects of teacher thinking and behaviour. Teachers’ confidence to use and teach various mathematics topics together with their beliefs about mathematics and learning had a marked effect on student learning and mathematical achievement. Beswick et al. suggest that the existence of horizontal content knowledge involves knowing how “current” teacher choices of their knowledge and practice may “facilitate or obstruct” the future learning of their students (p. 2). Vale, McAndrew and Krishnan (2011) highlight the positive impact of knowledge of mathematics on the horizon, knowing where students’ learning of mathematics is heading in the high school context. One teacher stated, “I feel confident in what I am doing... I can identify where they are going with their learning and having that knowledge informs what I am teaching them...” (p. 206). A growing body of evidence supports the need for knowledge at the mathematical horizon, or horizon content knowledge. How teachers develop this knowledge is the subject of ongoing research, and professional learning.

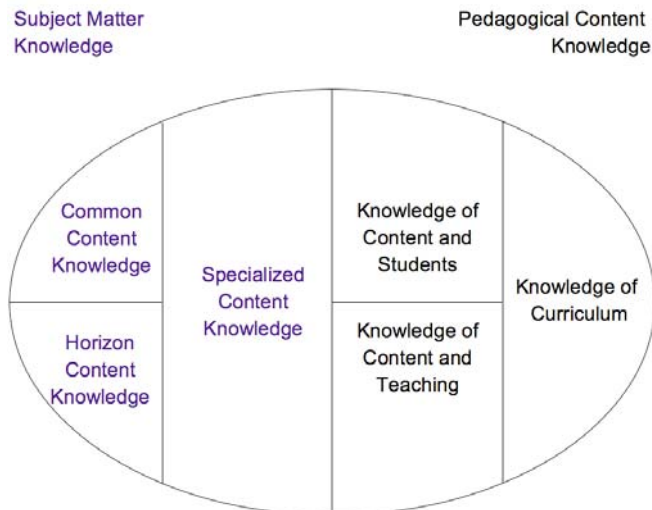


Figure 1. Mathematical knowledge for teaching (Ball et al., 2008, p. 403).

### *Professional Learning*

There is consensus from teachers, researchers, educators and educational bodies that teacher knowledge needs ongoing measurement, refinement and revision so as to understand the many demands of teaching. However, the ongoing process of the professional development for teachers needs to provide, not only theoretical knowledge about teaching and what to teach, but a strong connection to practice (Neubrand & Seago, 2009 p. 211). Clarke and Hollingsworth (2002) argue that professional development programs in the past were based on a ‘deficit - training - mastery’ model. Teachers are “relatively passive participants”, and programs frequently fail to consider the actual process of teacher learning and what constitutes *teacher development and change* (p. 948). Guskey (2002), for example, suggested that opportunities for learning and growth be embedded within a teacher’s everyday professional life and practice, with the most effective learning experiences and processes of change found within their own classroom.

Studies of professional learning have focused on teacher content knowledge (e.g., Vale et al., 2011) and specialised content knowledge (Bray, 2011), drawing their data from a range of sources (e.g., teacher logs, observations, interviews). Hill, Rowan and Ball (2005), for example, investigated the links between teachers professional development and their students’ achievement using of teacher logs and end-of-year questionnaires and student test results. However, few studies have been conducted in the naturalistic environment of the classroom. Recent classroom based studies where teachers critically examining video footage of themselves teaching with a peer have been found to be a powerful method for creating deliberate reflection leading to changes in teaching approaches (Muir, 2010).

This research study focuses on the teacher within their classroom environment. The research explores teachers’ subject matter knowledge (i.e., common content, specialised content and horizon content knowledge) in the area of number sense. These findings will then be examined in regards to how this impacts on the acquisition of number sense by students identified to be ‘at risk’ of failure, low progress or mathematical difficulty during the first year of their school learning.

## The Study

### *Study Design*

Aware of the “idiosyncratic and individual nature of teacher professional growth” (Clarke & Hollingsworth, 2002, p. 947) a collective case study design, as shown in Figure 2, was employed in this study. It enabled each of the participating teachers to be considered separately, while exploring the impact of their knowledge and practice on the mathematics learning and achievements of children within their classroom. Although, the design enables the synthesis of the findings from the individual cases to gain an insight into the learning community as a whole, a comprehensive discussion of the learning community is not within the scope of this paper.

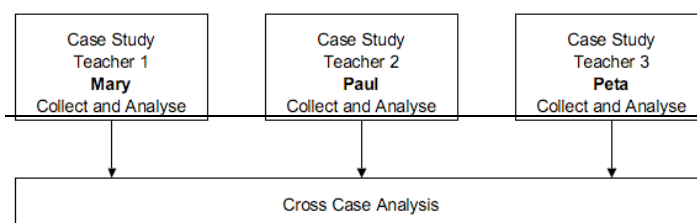


Figure 2. Collective case study design.

### *Greig Heights Primary School*

Three Kindergarten teachers from Greig Heights Primary School, situated in Western Sydney, who were working collaboratively in the development of their mathematics programs and content of their lessons, were recruited to participate in the study. Teacher 1 has over 30 years teaching experience with the last 10 teaching at Greig Heights. Teacher 2 is a beginning teacher teaching Kindergarten for the first time. Teacher 3 has been teaching for over 20 years and the last 7 years at Greig Heights.

Two of the classes were dedicated Kindergarten classes; the third class was a composite Kindergarten/ Year 1 class. Prior to commencing this study, students were assessed on the *Number Knowledge Test* or NKT (Griffin, 2005). Students identified at-risk were unable to complete correctly at least five of nine questions administered in Level 1. Eleven out of 18 (61%) students in Teacher 1’s class, six out of nine (67%) students in Teacher 2’s class, and eight out of sixteen (50%) students in Teacher 3’s class, were identified. These students were able to identify the smaller of two given numbers (e.g., “Which is smaller: 8 or 6?”), but had difficulties with understanding the magnitude of numbers and single digit subtraction (e.g., “How much is 8 take away 6?”). Students operating at this level are considered to still require concrete materials to understand quantity and yet to transition to from concrete to abstract. They are unable to use a “mental counting line” inside their heads and/or their fingers to keep track...” (Griffin, 2005, p. 273).

### *Case Study Implementation*

Each teacher was considered an individual case study; each case study was implemented in accordance with Figure 3. Prior to commencing the study, the first author met with the teachers to orientate them to the study. During the orientation, an overview and aims of the study were outlined. In addition, teachers were provided with an overview of number sense, and its importance with reference to recent research. Finally, a detailed explanation of the professional development cycle was presented to the teachers.

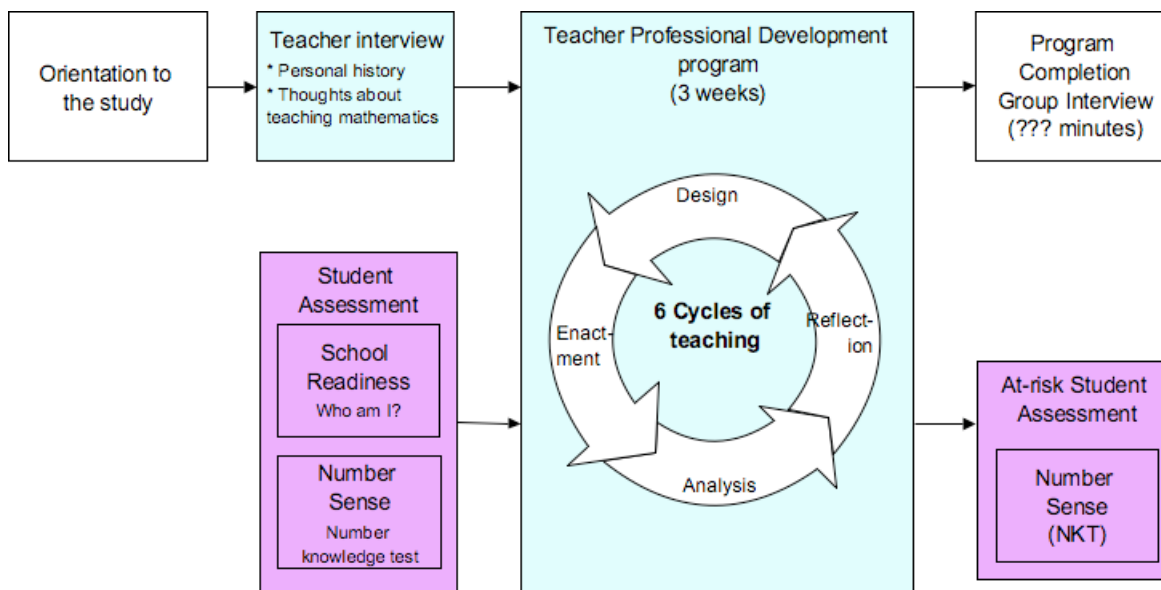


Figure 3. Overview of the study.

### *Teacher Professional Development*

The case study commenced with the first author interviewing each teacher to gain an understanding of their personal and teaching history, along with the participant's thoughts about teaching mathematics in their classroom. Following these interviews, each teacher taught six lessons over three weeks with a focus on developing number sense. The teaching of each lesson occurred within a cycle that enabled the collection of a range of data to capture the complexity of classroom teaching and learning, while allowing a process by which a teacher's professional growth may be enhanced in the context of students' acquisition of number sense. The cycle comprised four elements:

1. Design – teachers collaboratively planned number sense lessons that reflected the Early Stage 1 learning outcomes of the *Mathematics K-6 Syllabus* (Board of Studies NSW, 2002). During each lesson counting to 20 (NES1.1) and the process strand of working mathematically were incorporated. Other lesson content included:
  - Lesson 1, 2- Groups and shares collections of objects (NES1.3);
  - Lesson 3, 4 - Combines, separates and compares collections of objects (NES1.2), and
  - Lesson 5, 6 - Describing and recording halves, encountered in everyday contexts (NES1.4).
2. Enactment: teachers were observed and video-taped teaching each lesson within their classroom, with a focus of teacher practice and interactions with students identified *at-risk*. At the conclusion of each lesson the first author completed a set of field notes to record anecdotal evidence noted during the lesson.
3. Analysis: the first author, who acted as mentor to all three teachers, viewed each video observation and reviewed field notes.
4. Reflection: the teacher and first author met to discuss and reflect on the outcome of each lesson in terms of the teacher's mathematics knowledge, and pedagogical knowledge and practice. These reflections were used as a lead in further reflection and refinement of teacher knowledge and practice, with a focus on students identified at-risk.

## Results and Discussion

While the three teacher participants within this study presented as a strong collaborative team each brought with them variations in their awareness and insights into their role of teaching of mathematics, their professional knowledge and practice. An initial cross-case examination of the three teachers highlighted areas of convergence within the three areas of content knowledge.

### *Common Content Knowledge*

All teachers noted the starting point for identifying common content knowledge was the *K-6 Mathematics Syllabus* (Board of Studies NSW, 2002). Teachers referred to specific content from the syllabus (e.g., “counting to 30 by the end of the year”), and their need to continually refer to gain knowledge they needed to teach. Teachers reported, however, that their content knowledge of teachers had been heavily influenced by participation in the professional learning program, *Count Me in Too*.

Teachers noted that they relied on engagement with colleagues to enhance content knowledge. Teacher 3, for example, stated she had developed her content knowledge from observing other teachers, claiming that her teacher training did not include mathematics content knowledge. This same teacher alluded to breadth and depth of knowledge needed to work students. She stated that students come to school able to count, however “they [students] don’t know what they are doing or why they are doing it”. She talked further about the need for students to develop relational understanding, “why 2 comes after 1”.

### *Horizon Content Knowledge*

Each of the teachers revealed evidence of being aware of horizon content knowledge; initial data analysis indicates that this knowledge was embedded within their reflections, with teachers not being overtly aware of its function. Teacher 1, for example, recognised the need for numeracy in everyday life, particularly the mathematical domain of number (i.e., counting and estimation). Furthermore, she acknowledged that knowing what kids can do and “what they need to do next” was essential for teaching mathematics. Teacher 2, an early career teacher, when discussing his classroom practice reflected on his experience as a high school student. He noted he often thinks about what happens to maths in high school and the relevance of students’ early learning: “what did we need that for, but at the same time, we couldn’t have gone there if we didn’t have that, ... they are going to need it later”. This role of horizon knowledge was seen in his lesson videos. He was observed using specific language (e.g., “can you think of another way of doing this?”), prompting students to develop a deeper understanding on mathematics concepts, and hopefully be able to use this concept in future learning.

Teachers discussed how they struggled to grasp an understanding of where students were in their early mathematics development. This discussion viewed horizon knowledge in a different light to that noted by Ball et al. (2009), through examining where students had come from. Teacher 3, for example stated:

For the [kindergarten] kids that hadn’t picked it up you didn’t really know what to do with them. All teachers need that idea of a continuum and how they [students] develop so when they get a child that doesn’t know how to do something, they can look back and see the steps that they need to take to bring that child up...

She was concerned that teachers who don’t have an understanding of where kids have come from, in terms of knowledge development, are unable to address the specific learning needs

of that child. This view of content knowledge had specific implications for students identified as experiencing difficulty in developing an understanding of number sense.

### *Specialised Content Knowledge*

A specific focus of this study was the teaching of number sense. It was found through initial interviews, and observations, that the three teachers were not aware of the role of number sense on longer term mathematical learning. Teacher's skills in focusing on specialised content were varied. Teacher 2 had just commenced targeting students who are exhibiting difficulties in early number. He worked with students individually using knowledge gained from professional learning sessions from *Count Me in Too*. Teacher 1 talked about how teaching the whole class was easier for her compared to small group work, which was usually hands-on. She found that when tasks were hands-on, it was harder to identify students' learning needs and recognise their learning achievements. Teacher 3, demonstrated during class evidence of specialised content knowledge. During an interview, she reflected, "A child's number sense is related to their understanding of the numbers themselves, they can recognise numerals, whether they know of quantities of numbers... counting ... and representing the numbers using various hand-on materials."

### Conclusion

This cross-case examination of three Kindergarten teachers highlights the strength of obtaining direct classroom observations, alongside teacher reflections of their teaching, to gain an understanding of teacher knowledge. The breadth and depth of knowledge differed across teachers, and across the areas of content knowledge, specialised content knowledge and horizon content knowledge.

These levels of content knowledge have implications for all students, and the quality of learning they achieve. Teachers in this study used the required syllabus document to underpin common content knowledge. In these case studies, it was apparent that teachers were not aware of the role number sense (specialised content knowledge) plays in the development of early mathematics. Teachers provided evidence of the importance of horizon content knowledge, however, an understanding of where students have come from prior to arriving at school was seen as important. This understanding requires teachers to be able to identify misconceptions, and provide quality instruction that assists students acquire an understanding of key mathematical concepts. This latter understanding is key for students getting off to a good start in mathematics at school. Teachers who build key, conceptual links from existing knowledge to new, provide students with a base on which to become mathematically proficient (Kilpatrick, Swafford & Findell, 2001).

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